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**BAYESIAN RELIABILITY TEST PLANS
FOR
ONE-SHOT DEVICES**

by
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designing subsequent tests to meet threshold values of producer's and consumer's posterior risks.

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PREFACE

I accomplished this research under the sponsorship of the Rome Air Development Center while I was assigned to the Department of Mathematical Sciences at the United States Air Force Academy.

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SUMMARY

This report develops statistical reliability test plans for systems which are not continuously operating such as missiles, bombs, and fuses. The test plans are Bayesian in the sense that existing test data may be used to reduce the sample sizes required for subsequent testing. The prior probability density function of reliability is based upon this objective existing test data and is used to derive expressions for both producer's and consumer's posterior risks. Tables are provided for the practicing reliability manager to use in designing subsequent tests to meet threshold values of producer's and consumer's posterior risks.

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SECTION 1
BACKGROUND AND INTRODUCTION

Each year the Department of Defense spends about one-sixth of its budget on the research, development, testing and evaluation of new weapon systems. The reliability test programs associated with these systems are structured sequentially to qualify the designs functionally, demonstrate the operational capabilities of the systems under realistic environmental conditions, and prevent degradation in these operational capabilities during production and deployment. With rapidly rising costs and increasing complexities of hardware fabrication and testing, tremendous emphasis is being placed on using data from one test program to complement results of successive tests, thereby increasing the confidence associated with test conclusions. In this way the entire test program is viewed in a single thread continuum rather than as a series of separately conducted and reported events. Unfortunately, with the absence of an established scientific method for combining data from several test programs, past attempts to implement this approach have been necessarily subjective and largely discounted as "seat-of-the-pants."

Much work has been done in the reliability community to fill this void for continuously operating systems such as avionics, communications, electronics and radar. In particular, the inverted gamma probability distribution has been derived as a model for the Mean-Time-Between-Failure (MTBF) of such systems with exponentially distributed failure times. Data from preliminary tests may validly be used with the inverted gamma model to construct a baseline distribution on MTBF. Data from successive tests may then be used to

update this baseline. The confidence in estimating the system MTBF increases as testing progresses, eventually reaches an acceptable level, and testing ceases. This highly acclaimed approach is now complete with sets of tables available for the practicing reliability manager (Reference 1).

In this report a similar approach is developed for systems which are not continuously operating, such as missiles, bombs, and fuses. These "one-shot" devices have moved to the forefront in development priorities and are viewed as a primary means of enhancing the effectiveness of our weapon systems. For example, air combat exercises conducted at Nellis AFB in the late 1970's demonstrated that the operational effectiveness of the F-15 aircraft was severely restricted by the limitations of existing air-to-air missile systems. The \$8 billion Advanced Medium Range Air to Air Missile (AMRAAM) represents just one initiative to remove these limitations. Phrases like "launch and leave" and "multiple kills per pass" symbolize the resolve to improve our combat capability through the design of more effective armament packages. The results of this report make integrated test design and analysis possible for these systems.

SECTION 2
PROBLEM STATEMENT

The purpose of this report is to develop Bayesian reliability test plans for one-shot devices. The test plans will be Bayesian in the sense that relevant data from previous testing is to be used in attempt to reduce the sample size required in subsequent testing. From the standpoint of the user of such test plans, the problem may be stated in the following way.

"I need a test to demonstrate the reliability of a one-shot device — a device with a go or no-go function. The classic one-shot device is the cherry bomb; military examples include missiles, rocket motors, bombs and fuses. I have a certain reliability requirement and maximum allowable values for producer's risk α and consumer's risk β (risk is undefined at this point). Of course

$$0 \leq \alpha_m \leq 1$$
$$0 \leq \beta_m \leq 1$$

where the subscript m denotes the maximum allowable values of α and β . I have some test data from past tests which may be summarized as N_1 bernoulli trials and k_1 failures, and would like to use this data to reduce further testing. How many additional trials N_2 are required, and how many failures k_2 are allowed ($k_2 \leq M$) to do this?"

The answer to the problem, then, will be the number of trials required (N_2) and the maximum number of failures allowed (M) for an accept decision after the test is complete. The pair (N_2 , M) should satisfy the constraints

$$0 \leq \alpha \leq \alpha_m$$
$$0 \leq \beta \leq \beta_m$$

given the previous test result — N_1 trials and k_1 failures.

SECTION 3

RISK DEFINITION

Notice that the problem statement in Section 2 addresses maximum values of producer's and consumer's risks, but that producer's and consumer's risks were not defined. Some of the more popular choices for risk definition are shown in Table 1. The classical risk definitions are eliminated since the probability statements are conditioned on a single value of r . In this way the prior data are not applicable and neither is the Bayesian approach. The average risks are a version of classical risks with values "averaged" over a range of reliabilities. The most logical choice under a Bayesian scheme are the posterior risks, which may be interpreted in the following way. The producer's risk is the probability that a rejected system will have "high" reliability and the consumer's risk is the probability that an accepted system will have "low" reliability. "High" and "low" are defined by the thresholds r_α and r_β , respectively. This report uses posterior risks for both producer and consumer. The risk pair labelled "other" has been studied in reference 3 and found to have poor convergence properties in terms of finding (N_2, M) solutions. For an excellent interpretation of all these risks, refer to Reference 2. The definition of variables in this problem is now complete, as shown in Table 2.

	Producers Risk α	Consumers Risk β
1. Classical Risks	$P(\text{reject} r = r_\alpha)$	$P(\text{accept} r = r_\beta)$
2. Average Risks	$P(\text{reject} r \geq r_\alpha)$	$P(\text{accept} r \leq r_\beta)$
3. Posterior Risks	$P(r \geq r_\alpha \text{reject})$	$P(r \leq r_\beta \text{accept})$
4. Other	$P(\text{reject})$	$P(r \leq r_\beta \text{accept})$

Table 1: Risk Definitions

<u>Variable</u>	<u>Definition</u>	<u>Given or To be Calculated</u>
N_1	Number of Trials in Prior Test	Given
k_1	Number of Failures in Prior Test	Given
N_2	Number of Trials in New Test	To be Calculated
k_2	Number of Failures in New Test	—
M	Maximum Number of Failures Allowed in New Test for an Accept Decision	To be Calculated
α	Producer's Posterior Risk, $P(r \geq r_\alpha \text{Reject})$	To be Calculated
β	Consumer's Posterior Risk, $P(r \leq r_\beta \text{Accept})$	To be Calculated
α_m	Maximum Tolerable Value For Producer's Posterior Risk	Given
β_m	Maximum Tolerable Value For Consumer's Posterior Risk	Given
r_α	Reliability Threshold For α Calculation	Given
r_β	Reliability Threshold For β Calculation	Given

Table 2: Variable Definition

SECTION 4

DEVELOPMENT OF THE PRIOR DENSITY FUNCTION

In this section the prior probability density function (pdf) of reliability is developed. This prior pdf will summarize our knowledge of reliability after the event (N_1, k_1) but before the event (N_2, k_2) . Thus our prior pdf will be based upon the objective test data: N_1 trials and k_1 failures.

Before the event (N_1, k_1) has occurred, assume a uniform pdf on the random variable r , true reliability. We are saying that all values of r are equally likely in the absence of any other information. Thus

$$f(r) = \begin{cases} 1, & 0 \leq r \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Conduct an experiment: N_1 bernoulli trials resulting in k_1 failures. Denote this experimental result by E_1 . The pdf of r , given this test data, follows from Bayes Theorem

$$f(r|E_1) = \frac{P(E_1|r)f(r)}{P(E_1)} \quad (2)$$

where $f(r)$ comes directly from (1), $P(E_1|r)$ is a simple binomial pdf, and $P(E_1)$ comes from the law of total probability:

$$P(E_1) = \int_0^1 P(E_1|r)f(r)dr. \quad (3)$$

We have

$$\begin{aligned} P(E_1) &= \int_0^1 \frac{N_1!}{k_1!(N_1 - k_1)!} r^{N_1 - k_1} (1 - r)^{k_1} \cdot 1 \cdot dr \\ &= \frac{N_1!}{k_1!(N_1 - k_1)!} \int_0^1 r^{N_1 - k_1} (1 - r)^{k_1} dr \end{aligned}$$

The integral may be recognized as the beta function, giving us

$$= \frac{N_1!}{k_1!(N_1 - k_1)!} \left[\frac{(N_1 - k_1)!(k_1)!}{(N_1 + 1)!} \right] = \frac{1}{N_1 + 1}. \quad (4)$$

Our result in (4) will now be used in the denominator of (2) to develop our conditional pdf of r .

Therefore

$$\begin{aligned} f(r|E_1) &= \frac{P(E_1|r)f(r)}{P(E_1)} \\ &= \frac{\frac{N_1!}{k_1!(N_1 - k_1)!} (1 - r)^{k_1} r^{N_1 - k_1} \cdot 1}{\frac{1}{N_1 + 1}} \\ &= \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} (1 - r)^{k_1} r^{N_1 - k_1}, \quad 0 \leq r \leq 1 \end{aligned} \quad (5)$$

Clearly (5) is the pdf of true reliability r , given the experimental result E_1 . As the new test $(N_2, k_2 \leq M)$ is designed, then, we will use (5) as the prior pdf. To simplify the notation we will simply use $f(r)$ to denote this new prior

density function. Equation (5) may be recognized as a beta pdf with the following characterizations:

$$E(r) = \frac{N_1 - k_1 + 1}{N_2 + 2}; \text{ mode} = \frac{N_1 - k_1}{N_1} \quad (6)$$

$$\text{Var}(r) = \frac{(N_1 - k_1 + 1)(k_1 + 1)}{(N_1 + 3)(N_1 + 2)^2} \quad (7)$$

To summarize this section, the user with test data (N_1, k_1) now has the pdf of reliability and its mean and variance. Probability statements about the random variable r readily follow by integrating (5) over some interval of reliability values.

It is interesting to note that (5) may be used to generate classical confidence intervals on the proportion, r . Suppose a test consists of N^* trials and k^* failures. To construct classical lower confidence limits (α percentile) on r , let $N_1 = N^* - 1$ and $k_1 = k^*$ in (5) and compute the α percentile of $f(r)$. We are gaining a "degree of freedom" in the Bayesian approach ($N^* = N_1 + 1$) by originally assuming the uniform prior pdf on r . To construct classical upper confidence limits ($1 - \alpha$ percentile) on r , let $N_1 = N^* - 1$ and $k_1 = k^* - 1$ in (5) and compute the $1 - \alpha$ percentile of $f(r)$.

In finding the percentiles of (5) it is useful to note that

$$\int_0^{r_0} \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} (1 - r)^{k_1} r^{N_1 - k_1} dr$$

is equivalent to

$$\sum_{I=0}^{k_1} \frac{k_1 (N_1 + 1)! r_0^{N_1 + 1 - I} (1 - r_0)^I}{I!(N_1 + 1 - I)!}$$

SECTION 5

DEVELOPMENT OF PRODUCER'S RISK

$$\alpha = P(r \geq r_a | \text{Reject})$$

Producer's posterior risk may be interpreted as the proportion of rejected systems with reliability above some threshold (r_a). We begin with the definition

$$\alpha = P(r \geq r_a | \text{Reject})$$

$$= \int_{r_a}^1 f(r | \text{Reject}) dr. \quad (8)$$

Now Bayes' Theorem will be applied to find the integrand of (8) as follows:

$$f(r | \text{Reject}) = \frac{P(\text{Reject} | r) f(r)}{P(\text{Reject})} \quad (9)$$

The terms in the numerator of (9) pose no problems: $f(r)$ is the prior pdf (5) based on the test data (N_1, k_1); and $P(\text{Reject} | r)$ is the probability of more than M failures out of N_2 trials in the new test. This latter term may be found using the binomial pdf

$$P(\text{Reject} | r) = P(k_2 > M | r)$$

$$\begin{aligned} &= \sum_{k_2=M+1}^{N_2} \frac{N_2!}{k_2!(N_2 - k_2)!} r^{N_2 - k_2} (1 - r)^{k_2} \\ &= 1 - \sum_{k_2=0}^M \frac{N_2!}{k_2!(N_2 - k_2)!} r^{N_2 - k_2} (1 - r)^{k_2} \end{aligned} \quad (10)$$

The denominator of (9) may be found using the law of total probability:

$$\begin{aligned}
 P(\text{Reject}) &= \int_0^1 P(\text{Reject}|r)f(r)dr \\
 &= \int_0^1 \left[1 - \sum_{k_2=0}^M \frac{N_2!}{k_2!(N_2-k_2)!} r^{N_2-k_2} (1-r)^{k_2} \right] \frac{(N_1+1)!}{k_1!(N_1-k_1)!} r^{N_1-k_1} (1-r)^{k_1} dr \\
 &= 1 - \sum_{k_2=0}^M \frac{N_2!(N_1+1)!}{k_2!k_1!(N_2-k_2)!(N_1-k_1)!} \int_0^1 r^{N_1+N_2-k_1-k_2} (1-r)^{k_1+k_2} dr \\
 &= 1 - \sum_{k_2=0}^M \frac{N_2!(N_1+1)!}{k_2!k_1!(N_2-k_2)!(N_1-k_1)!} \frac{(N_1+N_2-k_1-k_2)!(k_1+k_2)!}{(N_1+N_2+1)!} \\
 &= 1 - \frac{N_2!(N_1+1)!}{k_1!(N_1-k_1)!(N_1+N_2+1)!} \sum_{k_2=0}^M \frac{(N_1+N_2-k_1-k_2)!(k_1+k_2)!}{k_2!(N_2-k_2)!} \quad (11)
 \end{aligned}$$

We now have each of the three terms required for equation (9), the integrand of equation (8). Notice that the integration in (8) is accomplished with respect to the variable r , which does not appear in our result for $P(\text{Reject})$. Because of this, we can integrate the numerator of (9) and then divide by $P(\text{Reject})$. For the integral we have

$$\begin{aligned}
 \int_{r_a}^1 P(\text{Reject}|r)f(r)dr &= \\
 &= \int_{r_a}^1 \left[1 - \sum_{k_2=0}^M \frac{N_2!}{k_2!(N_2-k_2)!} r^{N_2-k_2} (1-r)^{k_2} \right] \frac{(N_1+1)!}{k_1!(N_1-k_1)!} r^{N_1-k_1} (1-r)^{k_1} dr
 \end{aligned}$$

$$\begin{aligned}
& - \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} \int_{r_a}^1 r^{N_1 - k_1} \sum_{I=0}^{k_1} \binom{k_1}{I} (-1)^I r^I dr \\
& - \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} \sum_{k_2=0}^M \frac{N_2!}{k_2!(N_2 - k_2)!} \int_{r_a}^1 r^{N_1 + N_2 - k_1 - k_2} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} (-1)^I r^I dr \quad (12)
\end{aligned}$$

For the first half of equation (12) we proceed as follows:

$$\begin{aligned}
& \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} \sum_{I=0}^{k_1} \binom{k_1}{I} (-1)^I \left[\frac{r^{N_1 - k_1 + I + 1}}{N_1 - k_1 + I + 1} \right] \Big|_{r_a}^1 = \\
& \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} \sum_{I=0}^{k_1} \binom{k_1}{I} (-1)^I \frac{(1 - r_a^{N_1 - k_1 + I + 1})}{N_1 - k_1 + I + 1} \quad (13)
\end{aligned}$$

For the second half of equation (12) we proceed in a similar fashion:

$$\begin{aligned}
& \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} \sum_{k_2=0}^M \frac{N_2!}{k_2!(N_2 - k_2)!} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} (-1)^I \left[\frac{r^{N_1 + N_2 - k_1 - k_2 + I + 1}}{N_1 + N_2 - k_1 - k_2 + I + 1} \right] \Big|_{r_a}^1 \\
& - \frac{(N_1 + 1)!}{k_1!(N_1 - k_1)!} \sum_{k_2=0}^M \frac{N_2!}{k_2!(N_2 - k_2)!} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} (-1)^I \frac{(1 - r_a^{N_1 + N_2 - k_1 - k_2 + I + 1})}{N_1 + N_2 - k_1 - k_2 + I + 1} \quad (14)
\end{aligned}$$

Now we have all the terms needed for α , and the result is shown in Figure 1.

The notation $\binom{N}{x} = N!/[x!(N-x)!]$ has been used for brevity.

To summarize this section, we have found an expression for the producer's posterior risk (α) as a function of N_1 , k_1 , r_a , N_2 and M . Now we turn to consumer's posterior risk.

$$\alpha = P(r \geq r_\alpha | \text{Reject})$$

$$= N_1 \binom{N_1}{k_1} \sum_{I=0}^{k_1} \binom{k_1}{I} \binom{1-r}{I} \frac{(1-r)^{N_1-k_1+I+1}}{(N_1-k_1+I+1)} - N_1 \binom{N_1}{k_1} \sum_{k_2=0}^{N_1-k_1} \binom{N_1}{k_2} \binom{k_1+k_2}{k_1} \sum_{I=0}^{N_1+N_2-k_1-k_2+I+1} \frac{(-1)^I \binom{1-r}{I} \frac{(1-r)^{N_1+N_2-k_1-k_2+I+1}}{N_1+N_2-k_1-k_2+I+1}}$$

$$1 - N_1 \binom{N_1}{k_1} \sum_{k_2=0}^{N_1} \frac{\binom{N_2}{k_2}}{(N_1+N_2+1)(k_1+k_2)}$$

$$= \sum_{I=0}^{k_1} \binom{k_1}{I} \binom{1-r}{I} \frac{(1-r)^{N_1-k_1+I+1}}{(N_1-k_1+I+1)} - \sum_{k_2=0}^{N_1} \binom{N_2}{k_2} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} \frac{(-1)^I \frac{(1-r)^{N_1+N_2-k_1-k_2+I+1}}{N_1+N_2-k_1-k_2+I+1}}{N_1(N_1-k_1)}$$

$$\frac{1}{N_1(N_1-k_1)} - \sum_{k_2=0}^{N_1} \frac{\binom{N_2}{k_2}}{(N_1+N_2+1)(k_1+k_2)}$$

Figure 1: α

SECTION 6

DEVELOPMENT OF CONSUMER'S RISK

$$\beta = P(r \leq r_\beta | \text{Accept})$$

Consumer's posterior risk may be interpreted as the proportion of accepted systems with reliability below some threshold (r_β). Once again we begin with the definition

$$\beta = P(r \leq r_\beta | \text{Accept})$$

$$\begin{aligned} &= \int_0^{r_\beta} f(r | \text{Accept}) dr \\ &= \int_0^{r_\beta} \frac{P(\text{Accept} | r) f(r)}{P(\text{Accept})} dr \\ &= \frac{1}{P(\text{Accept})} \int_0^{r_\beta} P(\text{Accept} | r) f(r) dr \end{aligned} \tag{15}$$

For the integral in (15) we have

$$\begin{aligned} &\int_0^{r_\beta} \sum_{k_2=0}^M \binom{N_2}{k_2} r^{N_2 - k_2} (1-r)^{k_2} (N_1 + 1) \binom{N_1}{k_1} r^{N_1 - k_1} (1-r)^{k_1} dr \\ &= (N_1 + 1) \binom{N_1}{k_1} \sum_{k_2=0}^M \binom{N_2}{k_2} \int_0^{r_\beta} r^{N_1 + N_2 - k_1 - k_2} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} (-1)^I r^I dr \\ &= (N_1 + 1) \binom{N_1}{k_1} \sum_{k_2=0}^M \binom{N_2}{k_2} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} (-1)^I \frac{r_\beta^{N_1 + N_2 - k_1 - k_2 + I + 1}}{N_1 + N_2 - k_1 - k_2 + I + 1} \end{aligned} \tag{16}$$

For β we simply divide (16) by the complement of $P(\text{Reject})$, given by (11).

The results are shown in Figure 2.

To summarize this section, we now have an expression for the consumer's posterior risk (β) as a function of N_1 , k_1 , r_β , N_2 , and M .

$$\beta = P(r \leq r_\beta | \text{Accept})$$

$$= (N_1 + 1) \binom{N_1}{k_1} \sum_{k_2=0}^M \binom{N_2}{k_2} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} (-1)^I \frac{r_\beta}{N_1 + N_2 - k_1 - k_2 + I + 1}$$

$$\frac{(N_1 + 1) \binom{N_1}{k_1} \sum_{k_2=0}^M \binom{N_2}{k_2} \frac{1}{(N_1 + N_2 + 1) \binom{N_1 + N_2}{k_1 + k_2}}}{(N_1 + 1) \binom{N_1}{k_1} \sum_{k_2=0}^M \binom{N_2}{k_2} \frac{1}{(N_1 + N_2 + 1) \binom{N_1 + N_2}{k_1 + k_2}}}$$

$$\frac{\sum_{k_2=0}^M \binom{N_2}{k_2} \sum_{I=0}^{k_1+k_2} \binom{k_1+k_2}{I} (-1)^I \frac{r_\beta}{N_1 + N_2 - k_1 - k_2 + I + 1}}{\sum_{k_2=0}^M \frac{\binom{N_2}{k_2}}{(N_1 + N_2 + 1) \binom{N_1 + N_2}{k_1 + k_2}}}$$

Figure 2: g

SECTION 7

THE SEARCH FOR (N_2, M) SOLUTIONS

At this point we have expressions for α and β as functions of N_1 , k_1 , N_2 , M , r_α , and r_β . We can plot α and β versus N_2 to produce the families of curves as shown in figures 3 through 6. Each curve is drawn for a particular M , as labelled. Analytically our problem is to find the smallest value of M , and then the smallest value of N_2 , which will yield values of α and β below the predetermined thresholds α_m and β_m . The problem is summarized in Table 3 and amounts to finding the inverse functions of the α and β expressions in terms of M and N_2 . An iterative scheme will be required.

To develop an algorithm for finding (N_2, M) solutions we note from figures 3 through 6 that α is an increasing function of N_2 and a decreasing function of M , while β is a decreasing function of N_2 and an increasing function of M . We will begin with $M = 0$ and increase N_2 until $\beta < \beta_m$. This assures us of finding the fewest additional tests ("shortest test") necessary to satisfy the β threshold. At this point (N_2, M) we will check to see if $\alpha < \alpha_m$, in which case we have a solution. Otherwise we must increase M (to reduce the value of α) and start over. The result is an iterative scheme as described in Table 4.

To illustrate, consider the example where past data consists of 20 trials and 1 failure and we have $r_\alpha = .9$, $r_\beta = .9$ and $\alpha_m = \beta_m = .25$. (See Figure 3). Our first (N_2, M) candidate is $(6, 0)$ but this gives α off of our scale ($> .30$). Next is $(11, 1)$ with $\alpha = .30$. Incrementing m again by 1, we try $(18, 2)$ with $\alpha = .23$. Since $\alpha = .23 < \alpha_m = .25$ the pair $(18, 2)$ is the "shortest test" solution.

Given:	N_1 trials k_1 failures r_α and r_β α_m and β_m Expression for α (Figure 1) Expression for β (Figure 2)
Find:	The smallest M and then the smallest N_2
Such that:	$\alpha < \alpha_m$ and $\beta < \beta_m$

Table 3: Problem Statement

One final note is in order. The possibility exists for the prior test data (N_1, k_1) to be so "favorable" with respect to α_m and β_m that additional testing is not required. To identify these cases we realize that if no additional testing is to be performed then an accept decision is implied. Since $\alpha = P(r > r_\alpha / \text{Reject})$ we have $\alpha = 0$ necessarily. Also, since $\beta = P(r < r_\beta / \text{Accept})$ we have $\beta = P(r < r_\beta)$. Therefore, the cases where no additional testing is required are those which satisfy $P(r < r_\beta) < \beta_m$, and this check can be readily performed using the results of Section 4 for the prior pdf of r .

Appendix A is a listing of computer programs used to accomplish this iterative scheme. The programs are written in Basic and were run on a TRS 80 Model III Computer. The results are tabled in Appendix B.

1. Draw a horizontal line at $\beta = \beta_m$. The first β plot intersected (probably $M = 0$) is the shortest test to satisfy $\beta \leq \beta_m$.
2. Read the N_2 value for this test off the abscissa. We now have a candidate (N_2, M) solution.
3. Read the corresponding α value for this (N_2, M) pair. If $\alpha \leq \alpha_m$, the current (N_2, M) is adequate. Otherwise increase m by 1 and repeat the procedure. Be sure to keep $N_2 \geq M$ in all cases.

Table 4: Iterative Scheme

RISK PLOTS FOR RA = .9 : RB = .9 : N1 = 20 : N2 = 1

ALPHA

BETA

N1

N2

N1

N2

N1

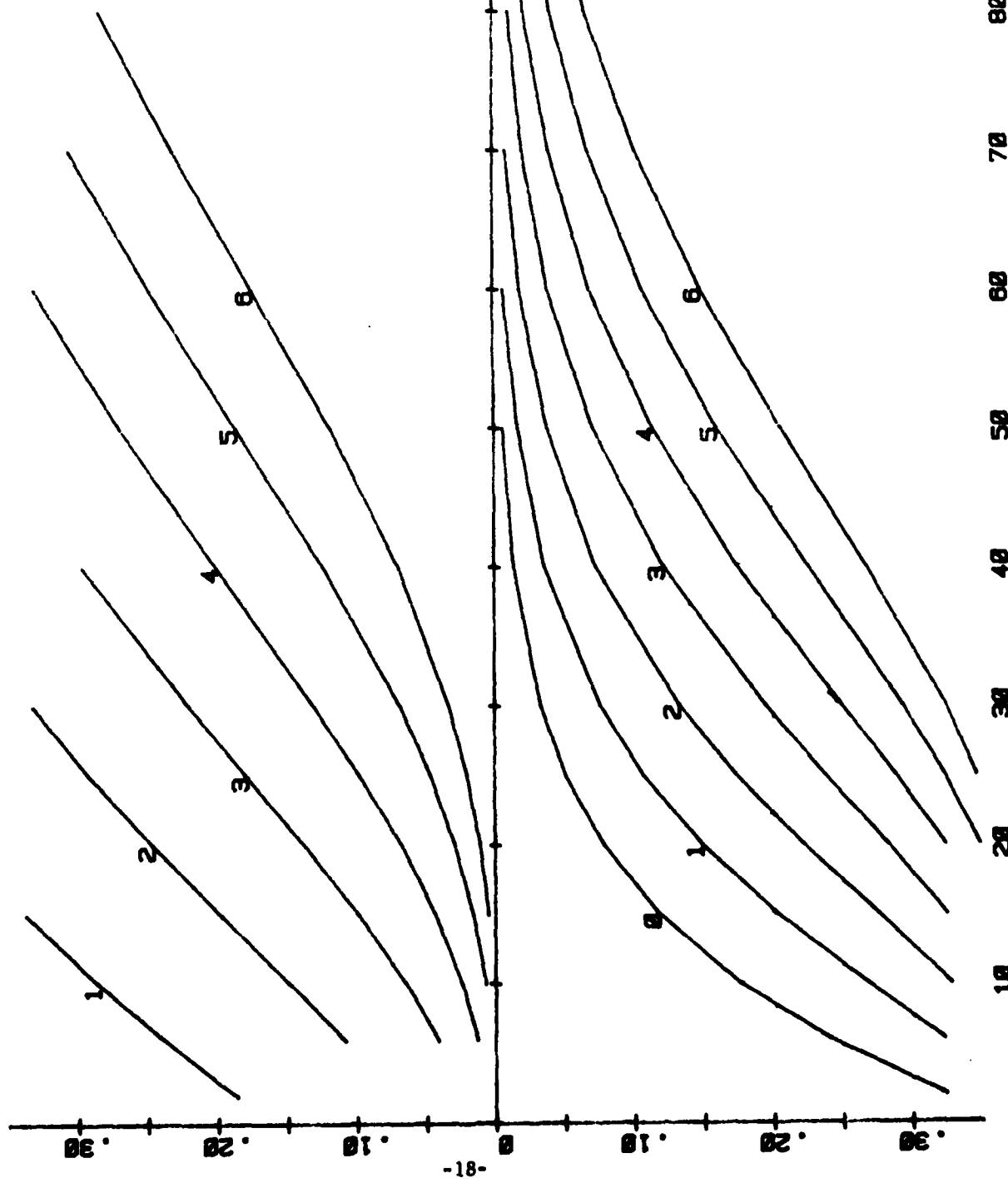
N2

N1

N2

N1

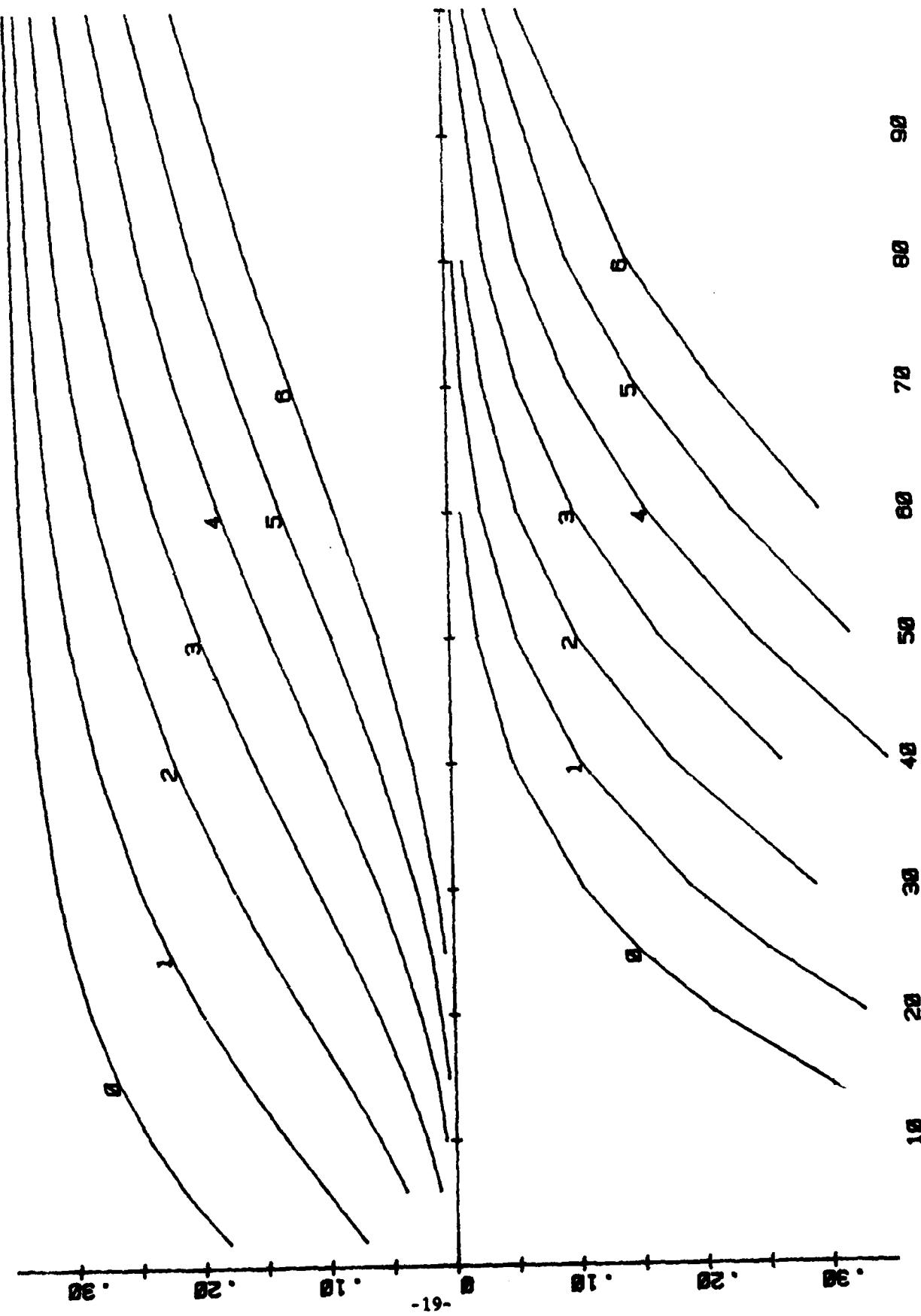
N2



RISK PLOTS FOR RA = .8, RB = .9, N1 = 20, K1 = 2

ALPHA

BETA



RISK PLOTS FOR RA = .8 , RB = .9 , N1 = 28 , K1 = 3

ALPHA

BETA

N2

99

89

79

69

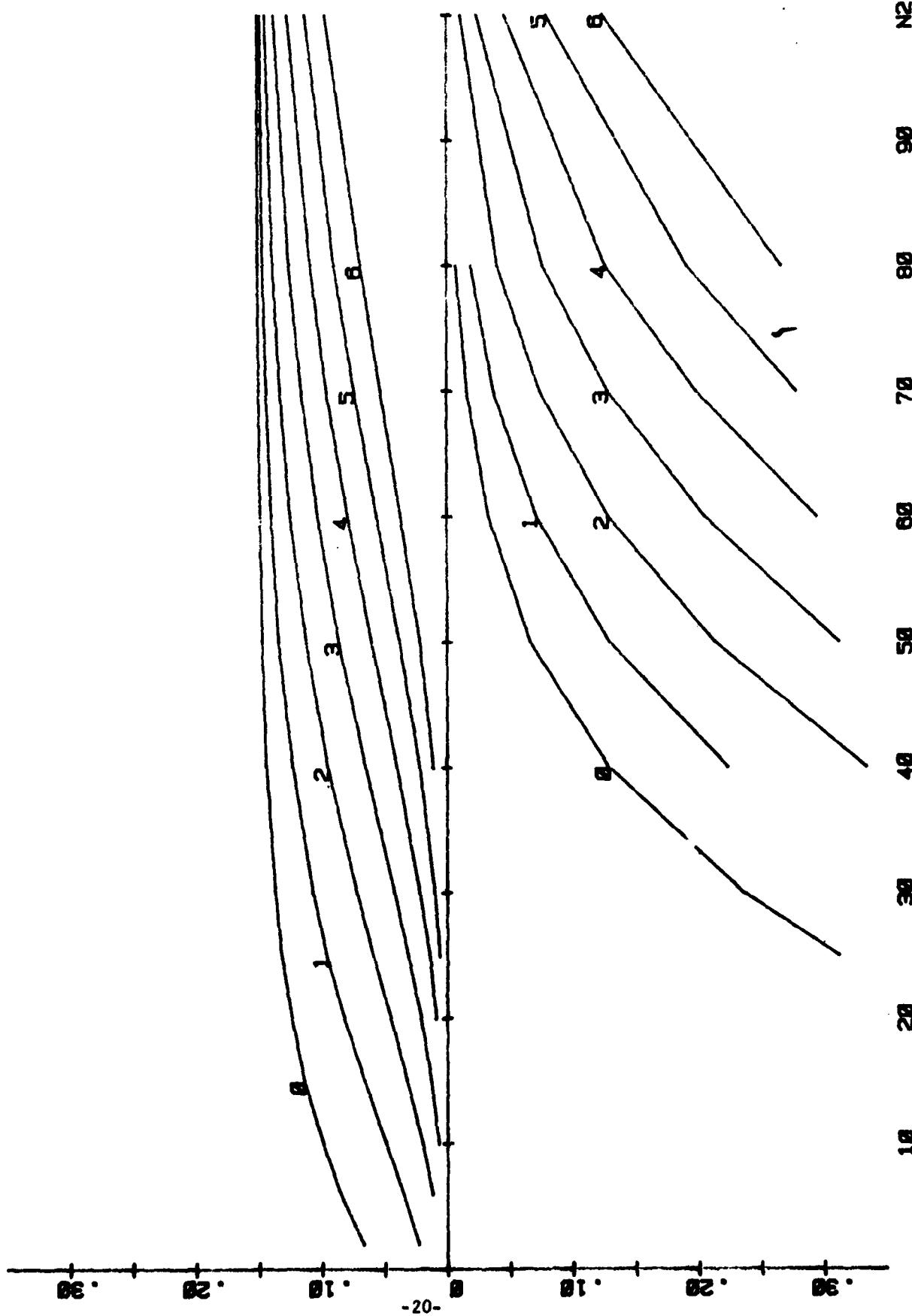
59

49

39

29

19



RISK PLOTS FOR $R_A = .8$, $R_B = .8$, $N_1 = 10$, $K_1 = 1$

ALPHA

BETA

N2

90

80

70

60

50

40

30

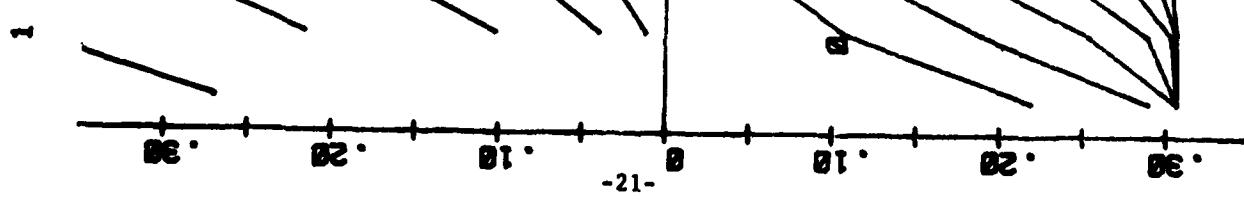
20

10

1

.38 .28 .18 .08 -.08 -.18 -.28 -.38

-21



SECTION 8
SUMMARY AND CONCLUSION

Tables are now available (Appendix B) for the design of statistical tests under the conditions outlined in Table 3. Each table is constructed for a given r_α , r_β , α_m , β_m and for a variety of (N_1, k_1) pairs. Tabled values are the minimum (N_2, M) required to keep $\alpha \leq \alpha_m$ and $\beta \leq \beta_m$. Note that when past test data (N_1, k_1) are favorable with respect to the reliability thresholds r_α and r_β , economies in sample size N_2 over classical tests result. It is noted, however, that "favorable" is somewhat difficult to define. If past data (N_1, k_1) are not favorable then corrective action is in order and Bayesian tests may not be appropriate. In these cases classical tests should be applied after implementation of corrective action.

APPENDIX A

COMPUTER PROGRAMS

```

190 DEFINT I-N
500 REM-----
510 REM EXECUTIVE RROUTINE
520 REM
530 GOSUB 1000      ' INIT
540 GOSUB 13000     ' INPUTS
550 FOR K=1 TO 31
560 N1=JN(K)
570 FOR K1=0 TO 7
580 IF K1>N1 THEN GOTO 640
590 IF N1<4 THEN GOTO 605
600 IF K1>(.8*N1) THEN GOTO 640
605 GOTO 15000      ' CHECK FOR P(R<RB)< BETA MAX
610 GOSUB 10000     ' (N2,M) SOLUTION
620 GOSUB 11000     ' SCREEN PRINT
630 GOSUB 12000     ' DISK PRINT
640 NEXT K1
650 NEXT K
660 CLOSE
670 END
1000 REM-----
1010 REM INITIALIZE
1020 REM
1050 DIM IC(50),JN(50)
1060 FOR I=1 TO 20
1070 JN(I)=I
1080 NEXT I
1090 FOR I= 21 TO 31
1100 READ JN(I)
1110 NEXT I
1120 DATA 25,30,35,40,45,50,60,70,80,90,100
1130 REM SET UP INTEGER COEFFS
1140 REM FOR SIMPSONS RULE
1150 REM
1155 ND=20          ' NUMBER OF SUBDIVISIONS (EVEN)
1160 IC(0)=1
1170 IC(ND)=1
1180 FF=2
1190 FOR I=1 TO (ND-1)
1200 FF=6-FF
1210 IC(I)=FF
1220 NEXT I
1230 RETURN
2000 REM-----
2010 REM SET UP FOR SIMPSONS RULE
2020 REM
2030 Z=5            ' NUMBER OF STANDARD DEVIATIONS
2050 RM=(N1-K1+1)/(N1+2)      ' MEAN OF BETA PRIOR
2060 RS=SQR(((K1+1)*(N1-K1+1))/((N1+3)*(N1+2)*(N1+2)))
2070 R0=RM-Z*RS        ' LOWER LIMIT
2080 IF R0<0 THEN R0=.00001
2090 R9=RM+Z*RS        ' UPPER LIMIT
2100 IF R9>1 THEN R9=.99999
2190 RETURN
3000 REM-----
3010 REM CALCULATE F(R) COEFFICIENT
3020 REM
3030 X1=1
3040 FOR I=(N1-K1+1) TO (N1+1)
3050 X1=X1*I
3060 NEXT I
3070 FOR I=1 TO K1
3080 X1=X1/I
3090 NEXT I

```

```

3100 RETURN
4000 REM-----
4010 REM                               CALCULATE F(R) GIVEN COEFFICIENT
4020 REM
4030 F=X1*RL*(N1-K1)*(1-R)*LK1
4040 RETURN
5000 REM-----
5010 REM                               CALCULATE P(ACC/R) & P(REJ/R)
5020 REM
5030 PA=R*IN2
5040 IF M=0 THEN GOTO 5100
5050 P1=PA
5060 FOR I=1 TO M
5070 P1=P1*(N2-I+1)*(1-R)/(R*I)
5080 PA=PA+P1
5090 NEXT I
5100 PR=1-PA
5110 IF PR<0 THEN PR=0:PA=1
5120 IF PR>1 THEN PR=1:PA=0
5130 RETURN
6000 REM-----
6010 REM                               APPLY SIMPSONS RULE (BETA)
6020 REM
6030 FS=0
6040 RD=(RU-RL)/ND
6050 FOR J=0 TO ND
6060 R=RL+J*RD
6070 GOSUB 5000
6080 GOSUB 4000
6090 FS=FS+IC(J)*PA*F
6100 NEXT J
6110 FS=FS*RD/3
6120 RETURN
7000 REM-----
7010 REM                               CALCULATE BETA
7020 REM
7030 IF RB<RL THEN B=0:GOTO 7090
7040 RL=R0:RU=R9:GOSUB 6000
7050 XDEN=FS
7060 RL=R0:RU=RB:GOSUB 6000
7070 XNUM=FS
7075 IF XDEN=0 THEN B=0:GOTO 7090
7080 B=XNUM/XDEN
7082 IF B>.99 THEN B=.99
7090 RETURN
8000 REM-----
8010 REM                               APPLY SIMPSONS RULE (ALPHA)
8020 REM
8030 FS=0
8040 RD=(RU-RL)/ND
8050 FOR J=0 TO ND
8060 R=RL+J*RD
8070 GOSUB 5000
8080 GOSUB 4000
8090 FS=FS+IC(J)*PR*F
8100 NEXT J
8110 FS=FS*RD/3
8120 RETURN
9000 REM-----
9010 REM                               CALCULATE ALPHA
9020 REM
9030 IF RA>RU THEN A=1:GOTO 9090
9040 RL=R0:RU=R9:GOSUB 8000
9050 XDEN=FS
9060 RL=RA:RU=R9:GOSUB 8000
9070 XNUM=FS

```

```

9075 IF XDEN=0 THEN A=0:GOTO 9090
9080 A=XNUM/XDEN
9090 RETURN
10000 REM-----
10010 REM           FIND (N2,M) SOLUTION
10020 REM
10030 GOSUB 2000      'INIT SIMPSONS RULE
10040 GOSUB 3000      'F(R) COEFF
10050 M=0:N2=1
10060 GOSUB 14000     'GET BETA SOLUTION
10100 GOSUB 9000      'ALPHA
10110 IF A<AM THEN GOTO 10140
10120 M=M+1
10130 IF M=N2 THEN N2=N2+1:GOTO 10060
10135 GOTO 10060
10140 RETURN
11000 REM-----
11010 REM           PRINT TO SCREEN
11020 REM
11030 IF IHF=1 THEN GOTO 11100
11040 CLS
11050 PRINT" RA      RB      N1      K1      N2      M      A      B"
11060 PRINT" ===    ===    ===    ===    ===    ==    ===    ==="
11070 A$= " .## .## ## ## ## ## .## .##"
11080 IHF=1
11090 POKE 16916,2
11100 PRINT USING A$;RA,RB,N1,K1,N2,M,A,B
11110 RETURN
12000 REM-----
12010 REM           DISK PRINT
12020 REM
12030 IF JPF=1 THEN GOTO 12060
12040 OPEN "0",1,$$
12050 PRINT#1,RA;RB;AM;BM
12060 PRINT#1,N1;K1;N2;M;A;B
12070 JPF=1
12080 RETURN
13000 REM-----
13010 REM           REQUEST INPUTS
13020 REM
13030 CLS
13040 INPUT"INPUT R ALPHA AND R BETA = ";RA,RB
13050 PRINT
13060 INPUT"INPUT ALPHA MAX AND BETA MAX = ";AM,BM
13070 PRINT
13080 INPUT"INPUT NAME OF DATA FILE ";S$
13090 S$=S$+":1"
13100 RETURN
14000 REM-----
14010 REM           FIND SMALLEST N2 FOR B<B MAX
14020 REM
14040 GOSUB 7000
14050 BH=B:NL=N2
14060 IF BH<BM THEN GOTO 14210
14070 N2=25
14080 GOSUB 7000
14090 BL=B:NH=N2
14100 IF BL<BM THEN GOTO 14130
14105 BH=BL:NL=N2
14110 N2=N2+25
14120 GOTO 14080
14130 NN=INT (NL+(NH-NL)*(BM-BH)/(BL-BH)+.5)
14135 IF NN=N8 THEN GOTO 14200
14140 N2=NN
14150 GOSUB 7000
14160 IF B<BM THEN NN=NN+1:BL=B

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14170 IF B>BM THEN NL=NN: BH=B
14185 N8=NN
14190 GOTO 14130
14200 IF B<BM THEN GOTO 14210
14205 N2=N2+1: GOSUB 7000
14206 GOTO 14200
14210 RETURN
15000 REM-----
15010 REM CHECK FOR P(R<RB)<BETA MAX
15020 REM
15030 T1=RBE(N1+1)
15040 CDF=T1
15050 IF K1=0 THEN GOTO 15110
15060 FOR I=1 TO K1
15070 T2=T1*(1-RB)*(N1+2-I)/(RB*I)
15080 CDF=CDF+T2
15090 T1=T2
15100 NEXT I
15110 IF CDF>BM THEN GOTO 610 ' RESUME
15120 A=0: B=CDF: N2=0: M=0
15130 GOTO 620

APPENDIX B

TABLES FOR BAYESIAN RELIABILITY TEST PLANS

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .7 R BETA = .7 ALPHA MAX = .1 BETA MAX = .1

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .7 R BETA = .7 ALPHA MAX = .2 BETA MAX = .2

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7
===== ===== ===== ===== ===== ===== ===== ===== =====

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .7 R BETA = .7 ALPHA MAX = .3 BETA MAX = .3

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .8 R BETA = .8 ALPHA MAX = .1 BETA MAX = .1

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .8 R BETA = .8 ALPHA MAX = .2 BETA MAX = .2

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .8 R BETA = .8 ALPHA MAX = .4 BETA MAX = .4

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .9 R BETA = .9 ALPHA MAX = .1 BETA MAX = .1

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	30/ 1	38/ 0						
N1 = 2:	47/ 3	37/ 0	49/ 0					
N1 = 3:	54/ 4	36/ 0	48/ 0	59/ 0				
N1 = 4:	61/ 5	35/ 0	47/ 0	58/ 0				
N1 = 5:	68/ 6	34/ 0	45/ 0	57/ 0	70/ 0			
N1 = 6:	75/ 7	51/ 2	44/ 0	56/ 0	69/ 0			
N1 = 7:	81/ 8	69/ 4	43/ 0	55/ 0	68/ 0	80/ 0		
N1 = 8:	88/ 9	87/ 6	42/ 0	54/ 0	67/ 0	79/ 0	91/ 0	
N1 = 9:	85/ 9	105/ 8	41/ 0	53/ 0	66/ 0	78/ 0	91/ 0	201/ 0
N1 = 10:	83/ 9	113/ 9	40/ 0	52/ 0	65/ 0	77/ 0	90/ 0	101/ 0
N1 = 11:	80/ 9	130/11	49/ 1	52/ 0	64/ 0	76/ 0	89/ 0	101/ 0
N1 = 12:	85/10	137/12	69/ 3	51/ 0	63/ 0	75/ 0	88/ 0	100/ 0
N1 = 13:	75/ 9	153/14	88/ 5	50/ 0	62/ 0	74/ 0	87/ 0	99/ 0
N1 = 14:	72/ 9	160/15	107/ 7	49/ 0	61/ 0	73/ 0	86/ 0	98/ 0
N1 = 15:	68/ 9	167/16	125/ 9	48/ 0	61/ 0	73/ 0	85/ 0	97/ 0
N1 = 16:	58/ 8	174/17	143/11	48/ 0	60/ 0	72/ 0	84/ 0	96/ 0
N1 = 17:	54/ 8	181/18	160/13	47/ 0	59/ 0	72/ 0	84/ 0	96/ 0
N1 = 18:	44/ 7	178/18	178/15	67/ 2	59/ 0	71/ 0	83/ 0	95/ 0
N1 = 19:	33/ 6	185/19	195/17	97/ 5	58/ 0	70/ 0	83/ 0	95/ 0
N1 = 20:	27/ 6	182/19	212/19	116/ 7	57/ 0	70/ 0	82/ 0	95/ 0
N1 = 25:	0/ 0	159/18	267/26	225/19	84/ 3	65/ 0	78/ 0	91/ 0
N1 = 30:	0/ 0	117/15	272/28	312/29	207/16	61/ 0	73/ 0	86/ 0
N1 = 35:	0/ 0	50/ 9	258/28	357/35	316/28	167/11	68/ 0	81/ 0
N1 = 40:	0/ 0	0/ 0	206/24	372/38	402/38	299/25	114/ 5	75/ 0
N1 = 45:	0/ 0	0/ 0	137/18	348/37	447/44	406/37	259/20	70/ 0
N1 = 50:	0/ 0	0/ 0	50/10	296/33	472/48	492/47	389/34	198/13
N1 = 60:	0/ 0	0/ 0	0/ 0	137/19	395/43	562/57	592/57	479/43
N1 = 70:	0/ 0	0/ 0	0/ 0	0/ 0	224/28	485/52	652/66	672/65
N1 = 80:	0/ 0	0/ 0	0/ 0	0/ 0	0/ 0	313/37	584/62	710/72
N1 = 90:	0/ 0	0/ 0	0/ 0	0/ 0	0/ 0	0/ 0	401/46	655/69
N1 = 100:	0/ 0	0/ 0	0/ 0	0/ 0	0/ 0	0/ 0	147/22	491/55

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = -.9 R BETA = -.9 ALPHA MAX = -.2 BETA MAX = -.2

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .9 R BETA = .9 ALPHA MAX = .3 BETA MAX = .44

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

K1 = 0 K1 = 1 K1 = 2 K1 = 3 K1 = 4 K1 = 5 K1 = 6 K1 = 7

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .45 R BETA = .95 ALPHA MAX = .1 BETA MAX = .1

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	43/ 0	200/ 0						
N1 = 2:	42/ 0	199/ 0	367/ 0					
N1 = 3:	66/ 1	198/ 0	366/ 0	533/ 0				
N1 = 4:	84/ 2	197/ 0	365/ 0	532/ 0				
N1 = 5:	98/ 3	196/ 0	364/ 0	531/ 0	698/ 0			
N1 = 6:	95/ 3	190/ 0	363/ 0	530/ 0	697/ 0			
N1 = 7:	110/ 4	145/ 0	362/ 0	529/ 0	696/ 0	864/ 0		
N1 = 8:	125/ 5	104/ 0	358/ 0	528/ 0	695/ 0	863/ 0	292/ 0	
N1 = 9:	123/ 5	88/ 0	228/ 0	527/ 0	694/ 0	862/ 0	1029/ 0	816/ 0
N1 = 10:	139/ 6	80/ 0	145/ 0	519/ 0	693/ 0	861/ 0	1028/ 0	303/ 0
N1 = 11:	137/ 6	93/ 1	119/ 0	269/ 0	692/ 0	860/ 0	1027/ 0	1195/ 0
N1 = 12:	152/ 7	107/ 2	107/ 0	178/ 0	659/ 0	859/ 0	1026/ 0	1194/ 0
N1 = 13:	151/ 7	105/ 2	100/ 0	147/ 0	283/ 0	858/ 0	1025/ 0	1193/ 0
N1 = 14:	165/ 8	121/ 3	96/ 0	132/ 0	203/ 0	523/ 0	1024/ 0	1192/ 0
N1 = 15:	163/ 8	139/ 4	92/ 0	124/ 0	172/ 0	286/ 0	1014/ 0	1191/ 0
N1 = 16:	161/ 8	156/ 5	90/ 0	119/ 0	156/ 0	222/ 0	425/ 0	1190/ 0
N1 = 17:	176/ 9	174/ 6	88/ 0	115/ 0	147/ 0	193/ 0	287/ 0	739/ 0
N1 = 18:	173/ 9	192/ 7	86/ 0	112/ 0	141/ 0	178/ 0	238/ 0	378/ 0
N1 = 19:	171/ 9	210/ 8	85/ 0	110/ 0	137/ 0	169/ 0	213/ 0	291/ 0
N1 = 20:	169/ 9	209/ 8	83/ 0	108/ 0	134/ 0	163/ 0	199/ 0	252/ 0
N1 = 25:	156/ 9	255/12	138/ 3	102/ 0	126/ 0	150/ 0	174/ 0	201/ 0
N1 = 30:	141/ 9	319/15	253/ 4	98/ 0	122/ 0	146/ 0	169/ 0	192/ 0
N1 = 35:	110/ 8	344/17	322/13	94/ 0	119/ 0	143/ 0	166/ 0	189/ 0
N1 = 40:	64/ 6	367/19	407/18	236/ 7	115/ 0	140/ 0	164/ 0	188/ 0
N1 = 45:	0/ 0	354/19	473/22	327/12	111/ 0	137/ 0	162/ 0	186/ 0
N1 = 50:	0/ 0	322/18	518/25	434/18	171/ 3	133/ 0	158/ 0	183/ 0
N1 = 60:	0/ 0	255/16	548/28	589/27	397/15	124/ 0	150/ 0	176/ 0
N1 = 70:	0/ 0	137/11	519/28	699/34	616/27	337/11	139/ 0	165/ 0
N1 = 80:	0/ 0	0/ 0	434/25	729/37	788/37	580/24	232/ 5	154/ 0
N1 = 90:	0/ 0	0/ 0	313/20	701/37	898/44	816/37	521/20	143/ 0
N1 = 100:	0/ 0	0/ 0	142/12	614/34	929/47	968/46	782/34	398/13

BAYESIAN RELIABILITY TEST PLAN

FOR

R ALPHA = .95 N BETA = .95 ALPHA MAX = .2 BETA MAX = .2

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	307/ 0	1627/ 0						
N1 = 2:	297/ 0	1617/ 0	3287/ 0					
N1 = 3:	287/ 0	1607/ 0	3277/ 0	4957/ 0				
N1 = 4:	277/ 0	1597/ 0	3267/ 0	4947/ 0				
N1 = 5:	267/ 0	1587/ 0	3257/ 0	4937/ 0	6607/ 0			
N1 = 6:	257/ 0	1217/ 0	3247/ 0	4927/ 0	6597/ 0			
N1 = 7:	247/ 0	717/ 0	3237/ 0	4917/ 0	6587/ 0	8267/ 0		
N1 = 8:	237/ 0	627/ 0	2977/ 0	4907/ 0	6577/ 0	8257/ 0	2737/ 0	
N1 = 9:	367/ 1	577/ 0	957/ 0	4847/ 0	6567/ 0	8247/ 0	9917/ 0	8167/ 0
N1 = 10:	357/ 1	547/ 0	827/ 0	3067/ 0	6557/ 0	8237/ 0	9907/ 0	2837/ 0
N1 = 11:	347/ 1	517/ 0	767/ 0	1137/ 0	6547/ 0	8227/ 0	9897/ 0	11577/ 0
N1 = 12:	327/ 1	497/ 0	737/ 0	1007/ 0	2057/ 0	8217/ 0	9887/ 0	11567/ 0
N1 = 13:	317/ 1	487/ 0	717/ 0	947/ 0	1287/ 0	8207/ 0	9877/ 0	11557/ 0
N1 = 14:	307/ 1	467/ 0	697/ 0	917/ 0	1177/ 0	1797/ 0	9867/ 0	11547/ 0
N1 = 15:	417/ 2	457/ 0	677/ 0	897/ 0	1127/ 0	1427/ 0	5117/ 0	11537/ 0
N1 = 16:	407/ 2	447/ 0	667/ 0	877/ 0	1097/ 0	1337/ 0	1777/ 0	11527/ 0
N1 = 17:	387/ 2	427/ 0	657/ 0	867/ 0	1077/ 0	1297/ 0	1577/ 0	2477/ 0
N1 = 18:	377/ 2	417/ 0	647/ 0	857/ 0	1067/ 0	1267/ 0	1497/ 0	1847/ 0
N1 = 19:	357/ 2	407/ 0	637/ 0	847/ 0	1057/ 0	1257/ 0	1467/ 0	1717/ 0
N1 = 20:	337/ 2	557/ 1	627/ 0	847/ 0	1047/ 0	1247/ 0	1447/ 0	1667/ 0
N1 = 25:	247/ 2	657/ 2	587/ 0	807/ 0	1017/ 0	1227/ 0	1417/ 0	1607/ 0
N1 = 30:	77/ 2	757/ 3	537/ 0	767/ 0	997/ 0	1207/ 0	1417/ 0	1607/ 0
N1 = 35:	07/ 0	677/ 3	667/ 1	727/ 0	967/ 0	1187/ 0	1407/ 0	1617/ 0
N1 = 40:	07/ 0	747/ 4	957/ 3	687/ 0	927/ 0	1157/ 0	1387/ 0	1607/ 0
N1 = 45:	07/ 0	647/ 4	1057/ 4	637/ 0	877/ 0	1117/ 0	1357/ 0	1587/ 0
N1 = 50:	07/ 0	537/ 4	1147/ 5	957/ 2	837/ 0	1077/ 0	1317/ 0	1557/ 0
N1 = 60:	07/ 0	07/ 0	1147/ 6	1357/ 5	737/ 0	977/ 0	1217/ 0	1457/ 0
N1 = 70:	07/ 0	07/ 0	957/ 6	1557/ 7	1177/ 3	877/ 0	1107/ 0	1347/ 0
N1 = 80:	07/ 0	07/ 0	517/ 5	1547/ 8	1577/ 6	957/ 1	1007/ 0	1237/ 0
N1 = 90:	07/ 0	07/ 0	07/ 0	1347/ 8	1957/ 9	1577/ 5	907/ 0	1137/ 0
N1 = 100:	07/ 0	07/ 0	07/ 0	937/ 7	1947/ 10	1977/ 8	1357/ 3	1027/ 0

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .95 R BETA = .45 ALPHA MAX = .3 BETA MAX = .3

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	22/ 0	64/ 0						
N1 = 2:	21/ 0	63/ 0	77/ 0					
N1 = 3:	20/ 0	62/ 0	76/ 0	90/ 0				
N1 = 4:	19/ 0	61/ 0	75/ 0	89/ 0				
N1 = 5:	18/ 0	60/ 0	74/ 0	88/ 0	102/ 0			
N1 = 6:	17/ 0	55/ 0	73/ 0	87/ 0	101/ 0			
N1 = 7:	16/ 0	48/ 0	72/ 0	86/ 0	100/ 0	114/ 0		
N1 = 8:	15/ 0	44/ 0	70/ 0	85/ 0	99/ 0	113/ 0	260/ 0	
N1 = 9:	14/ 0	42/ 0	63/ 0	84/ 0	98/ 0	112/ 0	126/ 0	816/ 0
N1 = 10:	13/ 0	40/ 0	60/ 0	82/ 0	97/ 0	111/ 0	125/ 0	270/ 0
N1 = 11:	12/ 0	38/ 0	58/ 0	77/ 0	96/ 0	110/ 0	124/ 0	138/ 0
N1 = 12:	11/ 0	37/ 0	57/ 0	75/ 0	94/ 0	109/ 0	123/ 0	137/ 0
N1 = 13:	10/ 0	36/ 0	56/ 0	74/ 0	91/ 0	108/ 0	122/ 0	136/ 0
N1 = 14:	9/ 0	34/ 0	55/ 0	73/ 0	90/ 0	106/ 0	121/ 0	135/ 0
N1 = 15:	8/ 0	33/ 0	54/ 0	72/ 0	89/ 0	105/ 0	120/ 0	134/ 0
N1 = 16:	7/ 0	32/ 0	53/ 0	72/ 0	89/ 0	105/ 0	119/ 0	133/ 0
N1 = 17:	6/ 0	31/ 0	52/ 0	71/ 0	89/ 0	105/ 0	119/ 0	133/ 0
N1 = 18:	12/ 1	30/ 0	51/ 0	71/ 0	88/ 0	105/ 0	120/ 0	133/ 0
N1 = 19:	11/ 1	29/ 0	50/ 0	70/ 0	88/ 0	105/ 0	120/ 0	134/ 0
N1 = 20:	9/ 1	28/ 0	49/ 0	69/ 0	88/ 0	105/ 0	121/ 0	135/ 0
N1 = 25:	0/ 0	23/ 0	45/ 0	66/ 0	87/ 0	105/ 0	123/ 0	139/ 0
N1 = 30:	0/ 0	30/ 1	40/ 0	63/ 0	84/ 0	105/ 0	124/ 0	142/ 0
N1 = 35:	0/ 0	24/ 1	36/ 0	58/ 0	81/ 0	102/ 0	123/ 0	144/ 0
N1 = 40:	0/ 0	17/ 1	31/ 0	54/ 0	76/ 0	99/ 0	121/ 0	143/ 0
N1 = 45:	0/ 0	8/ 1	40/ 1	49/ 0	72/ 0	94/ 0	117/ 0	140/ 0
N1 = 50:	0/ 0	0/ 0	34/ 1	44/ 0	67/ 0	89/ 0	112/ 0	136/ 0
N1 = 60:	0/ 0	0/ 0	33/ 2	34/ 0	57/ 0	79/ 0	102/ 0	125/ 0
N1 = 70:	0/ 0	0/ 0	9/ 2	52/ 2	47/ 0	69/ 0	91/ 0	113/ 0
N1 = 80:	0/ 0	0/ 0	0/ 0	38/ 2	52/ 1	59/ 0	81/ 0	103/ 0
N1 = 90:	0/ 0	0/ 0	0/ 0	20/ 2	55/ 2	49/ 0	71/ 0	92/ 0
N1 = 100:	0/ 0	0/ 0	0/ 0	0/ 0	55/ 3	70/ 2	61/ 0	82/ 0

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .98 R BETA = .98 ALPHA MAX = .1 BETA MAX = .1

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	113/ 0	491/ 0						
N1 = 2:	112/ 0	490/ 0	868/ 0					
N1 = 3:	111/ 0	489/ 0	867/ 0	1245/ 0				
N1 = 4:	109/ 0	488/ 0	866/ 0	1244/ 0				
N1 = 5:	108/ 0	487/ 0	865/ 0	1243/ 0	1621/ 0			
N1 = 6:	107/ 0	486/ 0	864/ 0	1242/ 0	1620/ 0			
N1 = 7:	106/ 0	484/ 0	863/ 0	1241/ 0	1619/ 0	1998/ 0		
N1 = 8:	176/ 1	483/ 0	861/ 0	1240/ 0	1618/ 0	1997/ 0	633/ 0	
N1 = 9:	173/ 1	482/ 0	860/ 0	1239/ 0	1617/ 0	1996/ 0	2374/ 0	816/ 0
N1 = 10:	232/ 2	481/ 0	859/ 0	1237/ 0	1616/ 0	1995/ 0	2373/ 0	742/ 0
N1 = 11:	226/ 2	480/ 0	858/ 0	1237/ 0	1615/ 0	1994/ 0	2372/ 0	2750/ 0
N1 = 12:	219/ 2	479/ 0	857/ 0	1235/ 0	1613/ 0	1993/ 0	2371/ 0	2749/ 0
N1 = 13:	266/ 3	477/ 0	856/ 0	1234/ 0	1612/ 0	1992/ 0	2370/ 0	2748/ 0
N1 = 14:	258/ 3	476/ 0	855/ 0	1233/ 0	1611/ 0	1991/ 0	2369/ 0	2747/ 0
N1 = 15:	251/ 3	473/ 0	854/ 0	1232/ 0	1611/ 0	1988/ 0	2366/ 0	2746/ 0
N1 = 16:	246/ 3	470/ 0	853/ 0	1232/ 0	1609/ 0	1987/ 0	2365/ 0	2745/ 0
N1 = 17:	241/ 3	466/ 0	852/ 0	1230/ 0	1608/ 0	1986/ 0	2364/ 0	2744/ 0
N1 = 18:	282/ 4	459/ 0	851/ 0	1229/ 0	1607/ 0	1986/ 0	2363/ 0	2741/ 0
N1 = 19:	278/ 4	450/ 0	850/ 0	1229/ 0	1607/ 0	1985/ 0	2364/ 0	2740/ 0
N1 = 20:	275/ 4	435/ 0	848/ 0	1228/ 0	1605/ 0	1984/ 0	2363/ 0	2739/ 0
N1 = 25:	307/ 5	260/ 0	825/ 0	1221/ 0	1601/ 0	1978/ 0	2356/ 0	2738/ 0
N1 = 30:	342/ 6	243/ 1	402/ 0	1205/ 0	1594/ 0	1974/ 0	2350/ 0	2731/ 0
N1 = 35:	375/ 7	312/ 3	274/ 0	547/ 0	1580/ 0	1967/ 0	2347/ 0	2723/ 0
N1 = 40:	407/ 8	348/ 4	240/ 0	347/ 0	668/ 0	1952/ 0	2340/ 0	2718/ 0
N1 = 45:	437/ 9	435/ 6	224/ 0	301/ 0	418/ 0	755/ 0	2322/ 0	2715/ 0
N1 = 50:	426/ 9	525/ 8	214/ 0	281/ 0	361/ 0	485/ 0	811/ 0	2687/ 0
N1 = 60:	401/ 9	651/11	299/ 2	263/ 0	325/ 0	393/ 0	477/ 0	603/ 0
N1 = 70:	374/ 9	768/14	491/ 6	252/ 0	312/ 0	371/ 0	433/ 0	502/ 0
N1 = 80:	343/ 9	837/16	679/10	244/ 0	304/ 0	362/ 0	420/ 0	479/ 0
N1 = 90:	274/ 8	859/17	854/14	288/ 1	297/ 0	356/ 0	414/ 0	471/ 0
N1 = 100:	201/ 7	879/18	976/17	541/ 6	290/ 0	351/ 0	410/ 0	467/ 0

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .98 R BETA = .98 ALPHA MAX = .2 BETA MAX = .2

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	78/ 0	456/ 0						
N1 = 2:	77/ 0	455/ 0	834/ 0					
N1 = 3:	76/ 0	454/ 0	833/ 0	1211/ 0				
N1 = 4:	75/ 0	453/ 0	832/ 0	1210/ 0				
N1 = 5:	74/ 0	452/ 0	831/ 0	1209/ 0	1587/ 0			
N1 = 6:	73/ 0	451/ 0	830/ 0	1208/ 0	1586/ 0			
N1 = 7:	72/ 0	450/ 0	829/ 0	1207/ 0	1585/ 0	1963/ 0		
N1 = 8:	71/ 0	449/ 0	827/ 0	1206/ 0	1584/ 0	1962/ 0	570/ 0	
N1 = 9:	70/ 0	448/ 0	826/ 0	1205/ 0	1583/ 0	1961/ 0	2340/ 0	816/ 0
N1 = 10:	69/ 0	447/ 0	825/ 0	1203/ 0	1582/ 0	1960/ 0	2339/ 0	645/ 0
N1 = 11:	68/ 0	445/ 0	824/ 0	1203/ 0	1581/ 0	1959/ 0	2338/ 0	2716/ 0
N1 = 12:	67/ 0	444/ 0	823/ 0	1201/ 0	1579/ 0	1958/ 0	2337/ 0	2715/ 0
N1 = 13:	66/ 0	442/ 0	822/ 0	1200/ 0	1578/ 0	1957/ 0	2336/ 0	2714/ 0
N1 = 14:	65/ 0	440/ 0	821/ 0	1199/ 0	1577/ 0	1956/ 0	2335/ 0	2713/ 0
N1 = 15:	64/ 0	435/ 0	819/ 0	1198/ 0	1577/ 0	1954/ 0	2332/ 0	2712/ 0
N1 = 16:	63/ 0	429/ 0	819/ 0	1197/ 0	1575/ 0	1953/ 0	2331/ 0	2711/ 0
N1 = 17:	62/ 0	420/ 0	817/ 0	1196/ 0	1574/ 0	1952/ 0	2330/ 0	2710/ 0
N1 = 18:	61/ 0	404/ 0	817/ 0	1194/ 0	1573/ 0	1952/ 0	2329/ 0	2707/ 0
N1 = 19:	60/ 0	373/ 0	815/ 0	1194/ 0	1573/ 0	1951/ 0	2329/ 0	2706/ 0
N1 = 20:	59/ 0	298/ 0	813/ 0	1193/ 0	1570/ 0	1950/ 0	2328/ 0	2705/ 0
N1 = 25:	90/ 1	153/ 0	760/ 0	1187/ 0	1566/ 0	1943/ 0	2321/ 0	2704/ 0
N1 = 30:	83/ 1	132/ 0	210/ 0	1154/ 0	1560/ 0	1939/ 0	2316/ 0	2696/ 0
N1 = 35:	77/ 1	121/ 0	182/ 0	264/ 0	1533/ 0	1933/ 0	2313/ 0	2689/ 0
N1 = 40:	102/ 2	113/ 0	171/ 0	229/ 0	313/ 0	1906/ 0	2305/ 0	2684/ 0
N1 = 45:	95/ 2	106/ 0	163/ 0	218/ 0	275/ 0	359/ 0	2271/ 0	2680/ 0
N1 = 50:	86/ 2	140/ 1	158/ 0	211/ 0	263/ 0	319/ 0	401/ 0	2617/ 0
N1 = 60:	69/ 2	169/ 2	148/ 0	203/ 0	254/ 0	303/ 0	352/ 0	406/ 0
N1 = 70:	47/ 2	157/ 2	139/ 0	196/ 0	249/ 0	299/ 0	347/ 0	394/ 0
N1 = 80:	0/ 0	182/ 3	130/ 0	188/ 0	244/ 0	297/ 0	347/ 0	394/ 0
N1 = 90:	0/ 0	168/ 3	164/ 1	180/ 0	238/ 0	293/ 0	346/ 0	396/ 0
N1 = 100:	0/ 0	188/ 4	196/ 2	172/ 0	231/ 0	288/ 0	343/ 0	396/ 0

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .98 R BETA = .98 ALPHA MAX = .3 BETA MAX = .3

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	58/ 0	436/ 0						
N1 = 2:	57/ 0	435/ 0	814/ 0					
N1 = 3:	56/ 0	434/ 0	813/ 0	1191/ 0				
N1 = 4:	55/ 0	433/ 0	812/ 0	1190/ 0				
N1 = 5:	54/ 0	432/ 0	811/ 0	1189/ 0	1567/ 0			
N1 = 6:	53/ 0	431/ 0	810/ 0	1188/ 0	1566/ 0			
N1 = 7:	52/ 0	430/ 0	809/ 0	1187/ 0	1565/ 0	1943/ 0		
N1 = 8:	51/ 0	429/ 0	807/ 0	1186/ 0	1564/ 0	1942/ 0	533/ 0	
N1 = 9:	50/ 0	428/ 0	806/ 0	1185/ 0	1563/ 0	1941/ 0	2320/ 0	816/ 0
N1 = 10:	49/ 0	426/ 0	805/ 0	1183/ 0	1562/ 0	1940/ 0	2319/ 0	589/ 0
N1 = 11:	48/ 0	425/ 0	804/ 0	1183/ 0	1561/ 0	1939/ 0	2318/ 0	2696/ 0
N1 = 12:	47/ 0	423/ 0	803/ 0	1181/ 0	1559/ 0	1938/ 0	2317/ 0	2695/ 0
N1 = 13:	46/ 0	421/ 0	802/ 0	1180/ 0	1558/ 0	1937/ 0	2316/ 0	2694/ 0
N1 = 14:	45/ 0	417/ 0	801/ 0	1179/ 0	1557/ 0	1936/ 0	2315/ 0	2693/ 0
N1 = 15:	44/ 0	411/ 0	799/ 0	1178/ 0	1557/ 0	1934/ 0	2312/ 0	2692/ 0
N1 = 16:	43/ 0	401/ 0	799/ 0	1177/ 0	1555/ 0	1933/ 0	2311/ 0	2691/ 0
N1 = 17:	42/ 0	382/ 0	797/ 0	1175/ 0	1554/ 0	1932/ 0	2310/ 0	2690/ 0
N1 = 18:	41/ 0	333/ 0	796/ 0	1174/ 0	1553/ 0	1932/ 0	2309/ 0	2687/ 0
N1 = 19:	40/ 0	160/ 0	795/ 0	1174/ 0	1553/ 0	1931/ 0	2309/ 0	2686/ 0
N1 = 20:	39/ 0	139/ 0	792/ 0	1173/ 0	1550/ 0	1930/ 0	2308/ 0	2685/ 0
N1 = 25:	34/ 0	109/ 0	188/ 0	1166/ 0	1546/ 0	1923/ 0	2301/ 0	2684/ 0
N1 = 30:	29/ 0	97/ 0	150/ 0	224/ 0	1539/ 0	1919/ 0	2296/ 0	2676/ 0
N1 = 35:	24/ 0	89/ 0	140/ 0	188/ 0	249/ 0	1912/ 0	2293/ 0	2669/ 0
N1 = 40:	19/ 0	83/ 0	134/ 0	180/ 0	224/ 0	273/ 0	2285/ 0	2664/ 0
N1 = 45:	14/ 0	77/ 0	129/ 0	176/ 0	219/ 0	259/ 0	300/ 0	2660/ 0
N1 = 50:	25/ 1	72/ 0	125/ 0	173/ 0	217/ 0	257/ 0	294/ 0	328/ 0
N1 = 60:	0/ 0	61/ 0	116/ 0	167/ 0	215/ 0	258/ 0	297/ 0	333/ 0
N1 = 70:	0/ 0	51/ 0	107/ 0	161/ 0	211/ 0	258/ 0	302/ 0	341/ 0
N1 = 80:	0/ 0	71/ 1	98/ 0	153/ 0	206/ 0	257/ 0	304/ 0	348/ 0
N1 = 90:	0/ 0	58/ 1	88/ 0	144/ 0	200/ 0	253/ 0	304/ 0	352/ 0
N1 = 100:	0/ 0	45/ 1	78/ 0	135/ 0	192/ 0	247/ 0	301/ 0	352/ 0

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .99 R BETA = .99 ALPHA MAX = .1 BETA MAX = .1

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	228/ 0	918/ 0						
N1 = 2:	227/ 0	917/ 0	1608/ 0					
N1 = 3:	226/ 0	916/ 0	1607/ 0	2298/ 0				
N1 = 4:	225/ 0	915/ 0	1606/ 0	2297/ 0				
N1 = 5:	224/ 0	914/ 0	1605/ 0	2296/ 0	2986/ 0			
N1 = 6:	222/ 0	913/ 0	1604/ 0	2295/ 0	2985/ 0			
N1 = 7:	221/ 0	912/ 0	1603/ 0	2294/ 0	2984/ 0	3675/ 0		
N1 = 8:	220/ 0	911/ 0	1601/ 0	2293/ 0	2983/ 0	3674/ 0	3303/ 0	
N1 = 9:	219/ 0	910/ 0	1600/ 0	2292/ 0	2982/ 0	3673/ 0	4364/ 0	816/ 0
N1 = 10:	218/ 0	908/ 0	1599/ 0	2289/ 0	2981/ 0	3672/ 0	4363/ 0	4632/ 0
N1 = 11:	217/ 0	907/ 0	1598/ 0	2290/ 0	2980/ 0	3671/ 0	4362/ 0	5052/ 0
N1 = 12:	216/ 0	906/ 0	1597/ 0	2287/ 0	2977/ 0	3670/ 0	4361/ 0	5051/ 0
N1 = 13:	215/ 0	906/ 0	1595/ 0	2286/ 0	2976/ 0	3669/ 0	4360/ 0	5050/ 0
N1 = 14:	213/ 0	905/ 0	1596/ 0	2285/ 0	2975/ 0	3668/ 0	4359/ 0	5049/ 0
N1 = 15:	212/ 0	903/ 0	1593/ 0	2284/ 0	2976/ 0	3664/ 0	4354/ 0	5048/ 0
N1 = 16:	359/ 1	902/ 0	1593/ 0	2284/ 0	2973/ 0	3663/ 0	4353/ 0	5047/ 0
N1 = 17:	356/ 1	901/ 0	1591/ 0	2281/ 0	2972/ 0	3662/ 0	4352/ 0	5046/ 0
N1 = 18:	353/ 1	900/ 0	1591/ 0	2280/ 0	2971/ 0	3664/ 0	4351/ 0	5041/ 0
N1 = 19:	349/ 1	898/ 0	1590/ 0	2281/ 0	2972/ 0	3663/ 0	4353/ 0	5040/ 0
N1 = 20:	346/ 1	898/ 0	1588/ 0	2280/ 0	2968/ 0	3662/ 0	4352/ 0	5039/ 0
N1 = 25:	448/ 2	891/ 0	1584/ 0	2273/ 0	2966/ 0	3653/ 0	4344/ 0	5042/ 0
N1 = 30:	520/ 3	884/ 0	1577/ 0	2268/ 0	2958/ 0	3651/ 0	4338/ 0	5033/ 0
N1 = 35:	579/ 4	871/ 0	1572/ 0	2262/ 0	2953/ 0	3643/ 0	4337/ 0	5023/ 0
N1 = 40:	557/ 4	842/ 0	1566/ 0	2259/ 0	2950/ 0	3638/ 0	4328/ 0	5018/ 0
N1 = 45:	628/ 5	769/ 0	1558/ 0	2251/ 0	2942/ 0	3635/ 0	4323/ 0	5017/ 0
N1 = 50:	617/ 5	607/ 0	1543/ 0	2246/ 0	2937/ 0	3627/ 0	4317/ 0	5008/ 0
N1 = 60:	685/ 6	511/ 1	1169/ 0	2229/ 0	2926/ 0	3616/ 0	4307/ 0	4997/ 0
N1 = 70:	753/ 7	632/ 3	597/ 0	1860/ 0	2912/ 0	3606/ 0	4296/ 0	4987/ 0
N1 = 80:	817/ 8	699/ 4	498/ 0	778/ 0	2536/ 0	3592/ 0	4286/ 0	4976/ 0
N1 = 90:	796/ 8	872/ 6	458/ 0	628/ 0	954/ 0	3007/ 0	4272/ 0	4966/ 0
N1 = 100:	854/ 9	1051/ 8	434/ 0	575/ 0	756/ 0	1115/ 0	3088/ 0	4952/ 0

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .99 R BETA = .99 ALPHA MAX = .2 BETA MAX = .2

(VALUES SHOWN ARE NUMBER OF TRIALS. MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	159/ 0	849/ 0						
N1 = 2:	158/ 0	848/ 0	1539/ 0					
N1 = 3:	157/ 0	847/ 0	1538/ 0	2229/ 0				
N1 = 4:	156/ 0	846/ 0	1537/ 0	2228/ 0				
N1 = 5:	154/ 0	845/ 0	1536/ 0	2227/ 0	2917/ 0			
N1 = 6:	153/ 0	844/ 0	1535/ 0	2226/ 0	2916/ 0			
N1 = 7:	152/ 0	843/ 0	1534/ 0	2225/ 0	2915/ 0	3606/ 0		
N1 = 8:	151/ 0	842/ 0	1532/ 0	2224/ 0	2914/ 0	3605/ 0	2495/ 0	
N1 = 9:	150/ 0	841/ 0	1531/ 0	2223/ 0	2913/ 0	3604/ 0	4295/ 0	816/ 0
N1 = 10:	149/ 0	839/ 0	1530/ 0	2220/ 0	2912/ 0	3603/ 0	4294/ 0	4633/ 0
N1 = 11:	148/ 0	838/ 0	1529/ 0	2221/ 0	2911/ 0	3602/ 0	4293/ 0	4983/ 0
N1 = 12:	147/ 0	837/ 0	1528/ 0	2218/ 0	2908/ 0	3601/ 0	4292/ 0	4982/ 0
N1 = 13:	146/ 0	837/ 0	1526/ 0	2217/ 0	2907/ 0	3600/ 0	4291/ 0	4981/ 0
N1 = 14:	145/ 0	836/ 0	1527/ 0	2216/ 0	2906/ 0	3599/ 0	4290/ 0	4980/ 0
N1 = 15:	144/ 0	834/ 0	1524/ 0	2215/ 0	2907/ 0	3595/ 0	4285/ 0	4979/ 0
N1 = 16:	143/ 0	833/ 0	1524/ 0	2215/ 0	2904/ 0	3594/ 0	4284/ 0	4978/ 0
N1 = 17:	142/ 0	832/ 0	1522/ 0	2212/ 0	2903/ 0	3593/ 0	4283/ 0	4977/ 0
N1 = 18:	141/ 0	830/ 0	1522/ 0	2211/ 0	2902/ 0	3595/ 0	4282/ 0	4972/ 0
N1 = 19:	140/ 0	829/ 0	1521/ 0	2212/ 0	2903/ 0	3594/ 0	4284/ 0	4971/ 0
N1 = 20:	139/ 0	829/ 0	1519/ 0	2211/ 0	2899/ 0	3593/ 0	4283/ 0	4970/ 0
N1 = 25:	134/ 0	822/ 0	1515/ 0	2204/ 0	2897/ 0	3584/ 0	4275/ 0	4973/ 0
N1 = 30:	130/ 0	812/ 0	1508/ 0	2199/ 0	2889/ 0	3582/ 0	4269/ 0	4964/ 0
N1 = 35:	125/ 0	788/ 0	1502/ 0	2193/ 0	2884/ 0	3574/ 0	4268/ 0	4954/ 0
N1 = 40:	120/ 0	716/ 0	1496/ 0	2190/ 0	2881/ 0	3569/ 0	4259/ 0	4949/ 0
N1 = 45:	115/ 0	442/ 0	1486/ 0	2182/ 0	2873/ 0	3566/ 0	4254/ 0	4948/ 0
N1 = 50:	181/ 1	332/ 0	1457/ 0	2177/ 0	2868/ 0	3558/ 0	4248/ 0	4939/ 0
N1 = 60:	168/ 1	273/ 0	479/ 0	2153/ 0	2857/ 0	3547/ 0	4238/ 0	4928/ 0
N1 = 70:	155/ 1	247/ 0	377/ 0	640/ 0	2838/ 0	3537/ 0	4227/ 0	4918/ 0
N1 = 80:	206/ 2	230/ 0	347/ 0	475/ 0	811/ 0	3520/ 0	4217/ 0	4907/ 0
N1 = 90:	191/ 2	216/ 0	330/ 0	441/ 0	568/ 0	949/ 0	4200/ 0	4897/ 0
N1 = 100:	175/ 2	282/ 1	318/ 0	425/ 0	532/ 0	659/ 0	1033/ 0	4878/ 0

BAYESIAN RELIABILITY TEST PLANS

FOR

R ALPHA = .99 R BETA = .99 ALPHA MAX = .3 BETA MAX = .3

(VALUES SHOWN ARE NUMBER OF TRIALS, MAXIMUM NUMBER OF FAILURES)

	K1 = 0	K1 = 1	K1 = 2	K1 = 3	K1 = 4	K1 = 5	K1 = 6	K1 = 7
N1 = 1:	118/ 0	809/ 0						
N1 = 2:	117/ 0	808/ 0	1499/ 0					
N1 = 3:	116/ 0	807/ 0	1498/ 0	2188/ 0				
N1 = 4:	115/ 0	806/ 0	1497/ 0	2187/ 0				
N1 = 5:	114/ 0	805/ 0	1496/ 0	2186/ 0	2877/ 0			
N1 = 6:	113/ 0	804/ 0	1495/ 0	2185/ 0	2876/ 0			
N1 = 7:	112/ 0	802/ 0	1494/ 0	2184/ 0	2875/ 0	3566/ 0		
N1 = 8:	111/ 0	801/ 0	1491/ 0	2183/ 0	2874/ 0	3565/ 0	2022/ 0	
N1 = 9:	110/ 0	801/ 0	1490/ 0	2182/ 0	2873/ 0	3564/ 0	4254/ 0	816/ 0
N1 = 10:	109/ 0	799/ 0	1489/ 0	2180/ 0	2872/ 0	3563/ 0	4253/ 0	4632/ 0
N1 = 11:	108/ 0	798/ 0	1488/ 0	2180/ 0	2871/ 0	3562/ 0	4252/ 0	4943/ 0
N1 = 12:	107/ 0	797/ 0	1487/ 0	2177/ 0	2868/ 0	3561/ 0	4251/ 0	4942/ 0
N1 = 13:	106/ 0	796/ 0	1486/ 0	2176/ 0	2866/ 0	3560/ 0	4250/ 0	4941/ 0
N1 = 14:	105/ 0	795/ 0	1486/ 0	2175/ 0	2865/ 0	3559/ 0	4249/ 0	4940/ 0
N1 = 15:	104/ 0	793/ 0	1484/ 0	2174/ 0	2867/ 0	3555/ 0	4245/ 0	4939/ 0
N1 = 16:	103/ 0	792/ 0	1484/ 0	2175/ 0	2863/ 0	3553/ 0	4244/ 0	4938/ 0
N1 = 17:	102/ 0	791/ 0	1482/ 0	2172/ 0	2862/ 0	3552/ 0	4243/ 0	4937/ 0
N1 = 18:	101/ 0	790/ 0	1482/ 0	2171/ 0	2861/ 0	3554/ 0	4242/ 0	4932/ 0
N1 = 19:	100/ 0	789/ 0	1481/ 0	2172/ 0	2863/ 0	3553/ 0	4244/ 0	4931/ 0
N1 = 20:	99/ 0	788/ 0	1478/ 0	2171/ 0	2859/ 0	3552/ 0	4243/ 0	4930/ 0
N1 = 25:	94/ 0	781/ 0	1474/ 0	2163/ 0	2856/ 0	3544/ 0	4234/ 0	4933/ 0
N1 = 30:	89/ 0	768/ 0	1468/ 0	2158/ 0	2848/ 0	3542/ 0	4229/ 0	4923/ 0
N1 = 35:	84/ 0	729/ 0	1462/ 0	2153/ 0	2843/ 0	3533/ 0	4227/ 0	4914/ 0
N1 = 40:	79/ 0	403/ 0	1455/ 0	2149/ 0	2840/ 0	3528/ 0	4218/ 0	4909/ 0
N1 = 45:	74/ 0	258/ 0	1442/ 0	2142/ 0	2833/ 0	3526/ 0	4213/ 0	4908/ 0
N1 = 50:	69/ 0	228/ 0	1395/ 0	2136/ 0	2827/ 0	3518/ 0	4208/ 0	4898/ 0
N1 = 60:	59/ 0	199/ 0	313/ 0	2104/ 0	2816/ 0	3507/ 0	4197/ 0	4888/ 0
N1 = 70:	49/ 0	182/ 0	284/ 0	389/ 0	2793/ 0	3496/ 0	4187/ 0	4877/ 0
N1 = 80:	39/ 0	168/ 0	270/ 0	364/ 0	458/ 0	3475/ 0	4176/ 0	4867/ 0
N1 = 90:	29/ 0	156/ 0	259/ 0	353/ 0	439/ 0	524/ 0	647/ 0	4856/ 0
N1 = 100:	52/ 1	145/ 0	250/ 0	346/ 0	433/ 0	513/ 0	589/ 0	669/ 0

APPENDIX C

REFERENCES

REFERENCES

1. Coppola, Anthony, Bayesian Reliability Tests Made Practical, RADC-TR-81-106, 1 June 1981.
2. Goel, Amrit L. and Anand M. Joglekar, Reliability Acceptance Sampling Plans Based Upon Prior Distribution, RADC-TR-78-294, Vol II, September 1976.
3. Wood, Buddy B., Integrated Reliability Test Plans: The Attributes Case, Technical Report, Department of Mathematical Sciences, U.S. Air Force Academy, CO, September 1982.